ENLIGHTENMENT TO PERFECTION

UNIVERSITY OF NORTH BENGAL

Revised Syllabus and Examination Scheme

for

M. Sc.

in

MATHEMATICS

Under **CBCS**

(To be implemented from Session 2017-18)

Signature of HOD
Department of Mathematics, NBU

Structure of Syllabus for M.Sc. in Mathematics

<table>
<tr><th></th><th>COURSES</th><th>PAPERS</th><th>INSTRUCTION HRS./ WEEK</th><th>DURATION OF EXAM (HRS.)</th><th>MARKS</th><th>CREDITS</th></tr>
<tr><td rowspan="12">SEMESTER : I</td><td rowspan="7">CORE COURSES</td><td>C-I1T: Groups and Rings</td><td>3</td><td>2</td><td>50</td><td>2</td></tr>
<tr><td>C-I2T: Naïve Set Theory and Elements of Topology</td><td>3</td><td>2</td><td>50</td><td>2</td></tr>
<tr><td>C-I3T: Analysis of Several Variables</td><td>3</td><td>2</td><td>50</td><td>2</td></tr>
<tr><td>C-I4T: Complex Analysis-I</td><td>3</td><td>2</td><td>50</td><td>2</td></tr>
<tr><td>C-I5T: Real Analysis</td><td>3</td><td>2</td><td>50</td><td>2</td></tr>
<tr><td>C-I6T: Ordinary Differential Equations</td><td>3</td><td>2</td><td>50</td><td>2</td></tr>
<tr><td colspan="4">TOTAL</td><td>12</td></tr>
<tr><td rowspan="3">CONTINUING EVALUATION</td><td>CLASS TESTS:</td><td colspan="3">In each paper there will be two class tests, first one of 6 marks & the second of 7 marks.</td><td rowspan="2">4</td></tr>
<tr><td>VIVA-VOCE:</td><td colspan="3">An internal Viva-voce of 22 marks will be conducted.</td></tr>
<tr><td colspan="4">TOTAL</td><td>4</td></tr>
<tr><td colspan="5">TOTAL CREDIT IN SEMESTER-I</td><td>16</td></tr>
</table>

	COURSES	PAPERS	INSTRUCTION HRS./ WEEK	DURATION OF EXAM (HRS.)	MARKS	CREDITS
SEMESTER : II	CORE COURSES	**C-II1T**: Linear Algebra	3	2	50	2
		C-II2T: Point-Set Topology	3	2	50	2
		C-II3T: Differential Geometry	3	2	50	2
		C-II4T: Functional Analysis	3	2	50	2
		C-II5T: Abstract Measure Theory	3	2	50	2
		C-II6T: Partial Differential Equations	3	2	50	2
					TOTAL	12
	CONTINUING EVALUATION	**CLASS TESTS:**	In each paper there will be two class tests, first one of 6 marks & the second of 7 marks			4
		SEMINAR:	An extension of any course taught during semester-I & II need to be prepared for presentation before the teacher concerned and it will carry 22 marks.			
					TOTAL	4
					TOTAL CREDIT IN SEMESTER-II	16

	COURSES	PAPERS	INSTRUCTION HRS./ WEEK	DURATION OF EXAM (HRS.)	MARKS	CREDITS
SEMESTER : III	**ELECTIVE COURSES** (Any six may be opted by a student of Mathematics/ Some of the elective papers **MATH-IIIT7 , MATH-IIIT9, MATH-IIIT10** may be opted by a student of NBU having not only interests but a proper knowledge in Mathematics subject to the availability of Teachers and number of students opted for the elective paper(s))	**E-IIIT1**: Measurability and Integration in Abstract Spaces	3	2	50	2
		E-IIIT2: Elementary Number Theory	3	2	50	2
		E-IIIT3: Complex Analysis –II	3	2	50	2
		E-IIIT4: : Field Extension and Galois Theory	3	2	50	2
		E-IIIT5: Topological Groups	3	2	50	2
		E-IIIT6: Algebraic Topology	3	2	50	2
		E-IIIT7: Integral Equation and Integral Transform	3	2	50	2
		E-IIIT8: Differential Topology	3	2	50	2
		E-IIIT9: Theory of Approximation	3	2	50	2
		E-IIIT10: p-adic Analysis	3	2	50	2
		TOTAL (for Mathematics students)				12

	CONTINUING EVALUATION	**CLASS TESTS:**	In each paper there will be two class tests, first one of 6 marks & the second of 7 marks.	4
		PROJECT:	A review work of 22 marks based on a research topic need to be presented by the students.	
			TOTAL	4
			TOTAL CREDIT IN SEMESTER-III (for Mathematics students)	16

	COURSES	PAPERS	INSTRUCTION HRS./ WEEK	DURATION OF EXAM (HRS.)	MARKS	CREDITS
SEMESTER : IV	**ELECTIVE COURSES** (Any five may be opted by a student of Mathematics/ The elective paper **MATH-IVT2,** may be opted by a student of NBU having not only interests but a proper knowledge in Mathematics subject to the availability of Teachers and number of students opted for the elective paper)	**E-IVT1:** Signed Measure and Product Measure	3	2	50	2
		E-IVT2: Graph Theory	3	2	50	2
		E-IVT3: Topological Algebra	3	2	50	2
		E-IVT4: General theory of Integration	3	2	50	2
		E-IVT5: Modules and Number Theory	3	2	50	2
		E-IVT6: Advanced complex Analysis	3	2	50	2
		E-IVT7: Discrete Mathematics	3	2	50	2
		E-IVT8: Algebraic Geometry	3	2	50	2
		E-IVT9: Category Theory	3	2	50	2
				TOTAL(for Mathematics students)		10

<table>
<tr><td rowspan="6"></td><td rowspan="2">SOFT COURSE</td><td>S-IV6P: Numerical problem solving by computer Programming (PRACTICAL)</td><td>4</td><td>4(1+3)</td><td>50</td><td>2</td></tr>
<tr><td colspan="4" align="right">TOTAL</td><td>2</td></tr>
<tr><td rowspan="3">CONTINUING EVALUATION</td><td>CLASS TESTS:</td><td colspan="3">In each paper there will be two class tests, first one of 6 marks & the second of 7 marks</td><td rowspan="2">4</td></tr>
<tr><td>VIVA-VOCE:</td><td colspan="3">A Grand viva-voce of 22 marks will be conducted in presence of one/two external examiner(s).</td></tr>
<tr><td colspan="4" align="right">TOTAL</td><td>4</td></tr>
<tr><td colspan="5" align="right">TOTAL CREDIT IN SEMESTER-IV (for Mathematics student)</td><td>16</td></tr>
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- **Question Pattern for all but Paper S-IV6P:** Group-A: (3 Questions out of 5)×10=30, Group-B: (2 harder problems out of 3) ×5= 10 and Group-C: (5 moderate problems) ×2=10.

- **Question Pattern for the Paper S-IV6P:** Group-A(Credit 1): (5 questions out of 7 about programming and numerical analysis) ×5=25 and Group-B(Credit 1): (Practical Note Book: 5)+(Viva-voce: 5)+(Solving 3 numerical problems by computer programming) ×5=15.

Detailed Syllabus of PG Semester-I

Paper: C-I1T
Groups and Rings

Homomorphism of Groups, Isomorphism Theorems, Cayley's Theorem, Generalized Cayley's Theorem, Group Action, Conjugacy Relation, Class Equation, Cauchy's Theorem, Sylow's Theorems and applications.

Ring Homomorphism. Isomorphism Theorems, Ideals and Quotient Ring. Prime and irreducible elements. Maximal and Prime Ideals. Quotient Field of an Integral Domain. Prime Fields. Irreducible and Prime Elements in a Ring. Factorisation Domain, Unique Factorisation Domain, Principal Ideal Domain, Euclidean Domain, Ring of Polynomials.

References

1. David S. Dummit and Richard M. Foote, Abstract Algebra (3e), John Wiley and Sons.
2. Joseph R. Gallian, Contemporary Abstract Algebra, Narosa Publishing House.
3. John B. Fraleigh, A First Course in Abstract Algebra, Narosa Publishing House.
4. Michael Artin, Algebra, Prentice Hall.
5. Thomas Hungerford, Algebra, Springer GTM.
6. I.N. Herstein, Topics in Abstract Algebra, Wiley Eastern Limited.
7. D. S. Malik, J. N. Modrdeson, and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition, 1997.
8. J.J. Rotman, The Theory of Groups: An Introduction, Allyn and Bacon, Inc., Boston.

Paper: C-I2T
Naïve Set Theory and elements of Topology:

Axiom of choice and existence of choice function. Partially ordered set, linearly ordered set, well ordered set and product of the same kinds, Zorn's lemma, well ordering principle with special emphasis on Ordinal and Cardinal numbers.

Topological spaces, open and closed sets, basis and sub-basis, closure, interior and boundary of a set. Subspace topology. Continuous maps: properties and constructions; Pasting Lemma. Open and closed maps, Homeomorphisms. Product topology, Quotient topology and examples of Topological Manifolds. Countability and separation axioms: Urysohn's lemma, Tietze extension theorem and applications. Urysohn embedding lemma and metrization theorem for second countable spaces. Connected, path-connected and locally connected spaces. Lindelof and Compact spaces. Net, Filters.

<u>References:</u>

1. J. R. Munkres, Topology: a first course, Prentice-Hall (1975).
2. G.F. Simmons, Introduction to Topology and Modern Analysis, TataMcGraw-Hill (1963).
3. M.A. Armstrong, Basic Topology, Springer.
4. J. L. Kelley, General Topology, Springer-Verlag (1975).
5. J. Dugundji, Topology, UBS (1999).
6. Stephen Willard, General Topology, Dover (2004).
7. I. P. Natanson, Theory of functions of a real variable, Vol. II. (especially for Ordinal numbers)

Paper: C-I3T
Analysis of Several Variables

Topology of $\mathbb{R}^n$, $GL_n(\mathbb{R})$ etc. . Differentiability of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ and the derivative as a linear map. Determinant as mapping; its continuity and differentiability. Existence and meaningfulness of e^A and its continuity as well as differentiability (A is a real square matrix). Higher derivatives, Chain Rule, mean value theorem for differentiable functions, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier, Sard's theorem. Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e., product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustration with plenty of examples. Inverse and implicit function theorems. Picard's Theorem.

Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$. Line integrals, Surfaces in $\mathbb{R}^3$, Surface integrals, Integration of forms, Divergence, Gradient and Curl operations, Green's theorem, Gauss (Divergence) theorem and Stoke's theorem.

References

1. M. Spivak: Calculus on manifolds, Benjamin (1965).
2. W. Rudin: Principles of mathematical analysis, Mc Graw-Hill.
3. T. Apostol: Mathematical Analysis
4. Munkres, J., Analysis on Manifolds.
5. T. Apostol: Calculus (Vol 2), John Wiley.

Paper: C-I4T
Complex Analysis I

Function of a complex variable, concept of limit and continuity, Stereographic Projection, Sequences and series of functions, Analytic function and Power series, Power series as analytic function, exponential, trigonometric, logarithmic, inverse functions, Complex integration., Cauchy-Goursat Theorem (for convex region), Winding number or index of a curve, Cauchy's integral formula, Higher order derivatives, Morera's Theorem, Cauchy's inequality and Liouville's theorem, Doubly periodic entire function, The fundamental theorem of algebra, Zeros of analytic functions, Maximum modulus principle, Hadamard's three circle theorem, Taylor's theorem, Schwarz lemma, Laurent's series, Isolated singularities, Casoratti-weierstrass theorem.

References

1. H. A. Priestly, Introduction to Complex Analysis, Clarendon Press Oxford, 1990.
2. J. B. Conway, Functions of one Complex variable. Springer-Verlag. International Student Edition, Narosa Pub. House. 1980.
3. Liang-shin Hahn & Bernard Epstein, Classical Complex Analysis. Jones and Bartlett Pub. International London, 1996.
4. L. V. Ahlfors. Complex Analysis, McGraw-Hill.
5. S. Lang. Complex Analysis, Addison Wesley. 1970.
6. D. Sarason, Complex Function Theory , Hindustan Book Agency, Delhi, 1994.
7. E. Hille, Analytic Function Theory (2 vols) , Gonn & Co, 1959.
8. W.H.J. Fuchs, Topics in the Theory of Functions of one complex variable, D. Van Nostrand Co. , 1967.
9. C. Caratheodory. Theolry of ;Functions (2 vols) Chelsea Publishing Company, 1964.
10. M. Heins, Complex Function Theory. Academic Press, 1968.
11. Walter Rudin, Real and Complex Analysis, McGraw - Hill Book Co, 1966.
12. S. Saks and A. Zygmund, Analytic Functions, Monografie Matematyczne, 1952.
13. E. C. Titchmarsh, The Theory of Functions, Oxford Univ. Press, London.
14. W. A. Veech, A Second Course in Complex Aanlysis. W. A. Benjamin, 1967.
15. S. Ponnusamy, Foundations of Complex Analysis, Narosa Pub. House, 1997.

Paper: C-I5T
Real Analysis

Extended real numbers, Algebraic operations and convergence in extended real number systems. Lebesgue outer measure, Measurable sets , regularity, Measurable Functions, Borel and Lebesgue measurability.

References:

1. Fundamentals of Real Analysis, S K. Berberian, Springer.
2. Measure Theory and Integration, G. De Barra, New Age International Publ.
3. Real Analysis, H. L. Royden.
4. Principles of Mathematical Analysis, W. Rudin.
5. Lectures on Real Analysis, J. Yeh, World Sci.
6. *R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966*

Paper: C-I6T
Ordinary Differential Equations

Review of solution methods for first order as well as second order equations, Power Series methods with properties of Bessel functions and Legendré polynomials.

Existence and Uniqueness of Initial Value Problems: Picard's and Peano's Theorems, Gronwall's inequality, continuation of solutions and maximal interval of existence, continuous dependence.

Higher Order Linear Equations and linear Systems: fundamental solutions, Wronskian, variation of constants, matrix exponential solution, behaviour of solutions.

Boundary Value Problems for Second Order Equations: Green's function, Sturm comparisontheorems and oscillations, eigenvalue problems.

References

1. S. L. Ross, Differential Equations, 3rd Edn., Wiley India, 1984.
2. G.F. Simmons, Differential Equations with Applications and Historical Notes, Tata-McGrawHill 2003.
3. M.Brown, Differential Equations and Their Applications, Springer 1983.
4. W. Boyce and R. Diprima, Elementary Differential Equations and Boundary Value Problems.
5. G. Birhoff & G.C. Rofa Ordinary Differential Equations, Wily ,1978

Detailed Syllabus of PG Semester-II

Paper: C-II1T
Linear Algebra

Linear transformations, Algebra of linear transformations, Matrix representation of linear transformations. Change of Basis.

Annihilating polynomials, diagonal forms, triangular forms, Direct Sum Decompositions, Invariant Direct sums, The Primary Decomposition Theorem.

Jordan Blocks and Jordan forms. Rational Canonical Form, Generalized Jordan form over an arbitrary field.

Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Bilinear forms, Symmetric Bilinear forms, Skew - Symmetric Bilinear forms.

References

1. K. Hauffman and R. Kunz, Linear Algebra, Pearson Education (INDIA), 2003.
2. G. Strang, Linear Algebra And Its Applications, 4th Edition, Brooks/Cole, 2006.
3. S. Lang, Linear Algebra, Springer, 1989.
4. David S. Dummit and Richard M. Foote, Abstract Algebra (3e), John Wiley and Sons.
5. R. Gallian Joseph, Conte mporary Abstract Algebra, Narosa Publishing House.
6. Thomas Hungerford, Algebra, Springer GTM.
7. I.N. Herstein, Topics in Abstract Algebra, Wiley Eastern Limited.
8. D.S. Malik, J.M. Mordesen, M.K. Sen, Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.

Paper: C-II2T
Point-Set Topology

- **(A)** Different kinds of compactness and their identity in metric spaces.
- **(B)** Local compactness and Compactifications.
- **(C)** Paracompactness.
- **(D)** Covering Spaces and Uniform Spaces.

<u>References:</u>

1. J. R. Munkres, Topology: a first course, Prentice-Hall (1975).
2. G.F. Simmons, Introduction to Topology and Modern Analysis, TataMcGraw-Hill (1963).
3. M.A. Armstrong, Basic Topology, Springer.
4. J. L. Kelley, General Topology, Springer-Verlag (1975).
5. J. Dugundji, Topology, UBS (1999).
6. Stephen Willard, General Topology, Dover (2004).
7. I. P. Natanson, Theory of functions of a real variable, Vol. II. (especially for Ordinal numbers)

Paper: C-II3T
Differential Geomtry

Curvilinear Coordinates, Elementary arc length, Length of a vector, Angle between two non-null vectors, Reciprocal Base system, Intrinsic Differentiation, Parallel vector fields.
Geometry of space curves: Serret-Frenet formulae, Equation of Straight lines, Helix, Bertrand curve.

Quick recap of multivariate calculus, Inverse Function Theorem and Implicit Function Theorem.

Regular surfaces, differential functions on surfaces, the tangent plane and the differential maps between regular surfaces, the first fundamental form, normal fields and orientability.

Gauss map, shape operator, the second fundamental form, normal and principle curvatures, Gaussian and mean curvatures.

Geodesic, Exponential map, Parallel transport, Theorem of Egregium.

Geodesic curvature, Gauss-Bonnet Theorem for simple closed curve.

References:

1. Elementary Differential Geometry, Andrew Pressley, Springer, 2010.
2. Elementary Differential Geometry, Barrett O'Neill, Elsevier, 2006.
3. Elementary Differential Geometry, Christian Bär, Cambridge University Press, 2011.
4. Differential Geometry of Curves and Surfaces, Manfredo P. Do Carmo, Prentice-Hall, Inc., Upper Saddle River, New Jersey 07458, 1976.
5. A Text Book of Differential Geometry, U. C. De, Asian Books Pvt. Ltd, 2014.
6. An Introduction to Differential Geometry (with the use of tensor Calculus), Princeton University Press, 1940.

Paper: C-II4T
Functional Analysis

Normed linear spaces. Banach spaces and examples. Quotient space of normed linear spaces and its completeness, equivalent norms. Riesz Lemma, basic properties of finite dimensional normed linear spaces and compactness. Weak convergence and bounded linear transformations, normed linear spaces of bounded linear transformations, dual spaces with examples. Uniform boundedness theorem and some of its consequences. Open mapping and closed graph theorems. Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces. Reflexive spaces.

Inner product spaces. Hilbert spaces. Orthonormal sets. Bessel's inequality. Comlplete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem. Riesz representation theorem. Adjoint of an operator on a Hilbert space. Reflexivity of Hilbert spaces. Self-adjoint operators, Positive, projection, normal and unitary operators.

References:

1. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966.
2. N. Dunford and J. T. Schwartz, Linear Operators, Part I, Interscience, New York, 1958.
3. R. E. Edwards, Functional Analysis. Holt Rinehart and Winston, New York, 1965.
4. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
5. R. B. Holmes, Geometric Functional Analysis and its Applications, Springer-Verlag 1975.
6. L. V. Kantorovich and G. P. Akilov, Functional Analysis, Pergamon Press, 1982.

7. K. Kreyszig , Introductory Functional Analysis with Applications, John Wiley & Sons New York, 1978.
8. B. K. Lahi;ri, Elements of Functional Analysis, The World Press Pvt. Ltd. Calcutta, 1994.
9. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
10. L. A. Lustenik and V. J. Sobolev, Elements of Functional Analysis, Hindustan Pub. Corpn. N.Delhi 1971.
11. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw -Hill Co. New York , 1963.
12. A. E. Taylor, Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.
13. K. Yosida, Functional Analysis, 3rd edition Springer - Verlag, New York 1971.
14. J. B. Conway, A course in functional analysis, Springer-Verlag, New York 199

Paper: C-II5T
Abstract Measure Theory

Abstract measure spaces: σ-algebra of sets, limit of sequences of sets, Borel σ-algebra, measure on a σ-algebra, measurable space and measure space.

Borel and Lebesgue measurability of functions on **R.** cantor ternary set and Cantor-Lebesgue function. Completion of Measure Space.

References:

1. Fundamentals of Real Analysis, S K. Berberian, Springer.
2. Measure Theory and Integration, G. De Barra, New Age International Publ.
3. Real Analysis, H. L. Royden.
4. Principles of Mathematical Analysis, W. Rudin.
5. Lectures on Real Analysis, J. Yeh, World Sci.
6. R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966

Paper: C-II6T
Partial Differential Equations

Cauchy Problems for 1st Order Hyperbolic Equations, Method of Characteristics etc.

Classification of Second Order Partial Differential Equations: normal forms and characteristics.

Initial and Boundary Value Problems: Lagrange-Green's identity and uniqueness by energy methods.

Laplace equation: mean value property, weak and strong maximum principle, Green's function, Poisson's formula, Dirichlet's principle, existence of solution using Perron's method (without proof).

Heat equation: initial value problem, fundamental solution, weak and strong maximum principle and uniqueness results.

Wave equation: uniqueness, D'Alembert's method, method of spherical means and Duhamel's principle.

Methods of separation of variables for heat, Laplace and wave equations.

References:

1. S. L. Ross, Differential Equations, 3rd Edn., Wiley India, 1984.
2. DiBenedetto, Partial Differential Equations, Birkhaüser, 1995.
3. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, 1998.
4. I.N. Sneddon Elements of Partial Differential Equations McGrawHill 1986.
5. R. Churchil & J. Brown, Fourier Series & Boundary Value Problems.
6. R.C. McOwen , Partial Differential Equations (Pearson Edu.) 2003.

Detailed Syllabus of PG Semester-III

Paper: E-IIIT1
Measurability and Integration in abstract Spaces

Construction of measure by means of outer measure; regular outer measure and metric outer measure.
Completion of measure space
Integration on a measure space

References:

1. J. Yeh, Lectures on Real Analysis, World Scientific.
2. H. L. Royden, Real analysis, Macmillan Publishing Co., Inc. 4th Edition, 1993.
3. S. K. Berberian. Measure and integration. Chelsea Publishing Company, NY, 1965.
4. G. de Barra, Measure Theory and integration, Wiley Eastern Ltd. 1981.
5. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd. New Delhi.
6. Richard L. Wheeden and Antoni Zygmund, Measure and Integral: An Introduction to Real Analysis, Marcel Dekker Inc. 1977.
7. P.R. Halmos, Measure Theory. Van Nostrand. Princeton 1950.
8. K. R. Parthasarathy , Introduction to Probability and Measure, Macmillan Co. India Ltd.Delhi - 1977.
9. R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966.
10. Serge Lang. Aanlysis I & II, Addison-Wesley Publishing Co. Inc. 1967.
11. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi 1997

Paper: E-IIIT2
Elementary Number Theory

Division algorithm, Greatest common divisor, Euclidean algorithm, Diophantine equations . Fundamental Theorem of Arithmetic.

Congruences, Binary and Decimal Representations of integers, Chinese remainder theorem, Fermat's Little Theorem, Pseudoprimes, Euler's Theorem, Wilson's theorem, linear congruences, order of an integer modulo a prime, primitive roots for primes, quadratic residues, Legendre's Symbol and its properties, Law of Quadratic Reciprocity.

Arithmetic functions like Mobius function, Euler phi function, greatest integer function etc. Mobius inversion formula, Dirichlet's product of arithmetical functions, Dirichlet's inverse, The Mangoldt function, Multiplicative and Completely Multiplicative functions, Formal Power Series, The Bell series of an arithmetical function, Derivatives of an arithmetical function.

<u>References:</u>

1. David M. Burton, Elementary Number Theory, University of New Hampshire.
2. G.H. Hardy, and , E.M. Wrigh,. An Introduction to the Theory of Numbers (6th ed, Oxford University Press, (2008).
3. W.W. Adams and L.J. Goldstein, Introduction to the Theory of Numbers, 3rd ed., Wiley Eastern, 1972.
4. A. Baker, A Concise Introduction to the Theory of Numbers, Cambridge University Press, Cambridge, 1984.
5. I. Niven and H.S. Zuckerman, An Introduction to the Theory of Numbers, 4th Ed., Wiley, New York, 1980.
6. T.M. Apostol, Introduction to Analytic number theory, UTM, Springer, (1976).
7. J. W. S Cassel, A. Frolich, Algebraic number theory, Cambridge.
8. M Ram Murty, Problems in analytic number theory, springer.
9. M Ram Murty and Jody Esmonde, Problems in algebraic number theory, springer.

Paper: E-IIIT3
Complex Analysis II

Residues. Cauchy's residue theorem, Evaluation of integrals, Rouche's theorem, Meromorphic functions, The argument principle, inverse function theorem, Branches of many valued functions with special reference to arg z , log z and z^a., Riemann surfaces.

Bilinear transformations, their properties and classifications, Definitions and examples of Conformal mappings, Cross-Ratio, Principle of Symmetry.

Analytic continuation, Uniqueness of direct analytic continuation, Monodromy theorem, Analytic continuation via Reflection, Uniqueness of analytic continuation along a curve, Power series method of analytic continuation.

References

1. . H. A. Priestly, Introduction to Complex Analysis, Clarendon Press Oxford, 1990.
2. J. B. Conway, Functions of one Complex variable. Springer-Verlag. International Student Edition, Narosa Pub. House. 1980.
3. Liang-shin Hahn & Bernard Epstein, Classical Complex Analysis. Jones and Bartlett Pub. International London, 1996.
4. L. V. Ahlfors. Complex Analysis, McGraw-Hill.
5. S. Lang. Complex Analysis, Addison Wesley. 1970.
6. D. Sarason, Complex Function Theory , Hindustan Book Agency, Delhi, 1994.
7. Mark J. Ablowitz and A. S. Fokas, Complex Variables: Introduction and Applications, Cambridge Univ. Press, South Asian edn. 1998.
8. E. Hille, Analytic Function Theory (2 vols) , Gonn & Co, 1959.
9. W.H.J. Fuchs, Topics in the Theory of Functions of one complex variable, D. Van Nostrand Co. , 1967.
10. C. Caratheodory. Theolry of ;Functions (2 vols) Chelsea Publishing Company, 1964.
11. M. Heins, Complex Function Theory. Academic Press, 1968.
12. Walter Rudin, Real and Complex Analysis, McGraw - Hill Book Co, 1966.
13. S. Saks and A. Zygmund, Analytic Functions, Monografie Matematyczne, 1952.
14. E. C. Titchmarsh, The Theory of Functions, Oxford Univ. Press, London.
15. W. A. Veech, A Second Course in Complex Aanlysis. W. A. Benjamin, 1967.
16. S. Ponnusamy, Foundations of Complex Analysis, Narosa Pub. House, 1997.

Paper: E-IIIT4
Field Extension and Galois Theory

Field extension – Algebraic and transcendental Extensions. Separable and Inseparable extensions. Perfect fields, Artin's Theorem, Normal extensions. Splitting fields of a polynomial. Finite fields. Primitive elements, Primitive Element Theorem, Algebraically closed fields, Algebraic closure of a field and its existence.

Galois extensions. Galois Group of automorphisms and Galois Theory, Fundamental theorem of Galois theory. Solutions of polynomial equations by radicals. Insolvability of the general equation of degree 5 (or more) by radicals.

References

1. *M. Artin, Algebra, Perentice -Hall of India, 1991.*
2. *P.M. Cohn, Algebra, vols, I,II, & III, John Wiley & Sons, 1982, 1989, 1991.*
3. *N. Jacobson, Basic Algebra, vols. I & II, W. H. Freeman, 1980 (also published by Hindustan Publishing Company)*
4. *S. Lang. Algebra, 3rd edn. Addison-Weslley, 1993.*
5. *I.S. Luther and I.B.S. Passi, Algebra, Vol.III-Modules, Narosa Publishing House.*
6. *D. S. Malik, J. N. Modrdeson, and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition, 1997.*
7. *Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999*
8. *I. Stweart, Galois Theory, 2nd edition, Chapman and Hall, 1989.*
9. *J.P. Escofier, Galois theory, GTM Vol.204, Springer, 2001.*

Paper: E-IIIT5
Topological Groups

Definition and examples of topological groups, Topologies generated by characters, Pseudonorms and invariant pseudometrics in a group, Function spaces as topological groups, Transformation groups, Subgroups and direct product of topological groups, Quotients of topological groups, initial and final topologies, Separation axioms, closed subgroups, Metrizability of topological groups, Connectedness in topological groups, Group topologies determined by sequences. The Zariski topology and the Markov topology, The Markov topology of the symmetric group, Existence of Hausdorff group topologies, Extension of group topologies, Cardinal invariants of topological groups, Completeness and completion of topological groups, Compactness and local compactness in topological groups: examples, specific properties of compactness and local compactness, general properties (the open mapping theorem, completeness, etc.), compactness vs connectedness, Properties of $\mathbb{R}^n$and its subgroups, The closed subgroup of $\mathbb{R}^n$, elementary LCA groups and Kronecker's theorem, on the structure of compactly generated locally compact abelian groups.

Reference:

1. Dikran Dikranjan, Introduction to topological groups.
2. Seth Warner, Topological Rings, Elsevier Science Publishers.
3. Ivan F. Wilde, Topological Vector Spaces (Lecture notes).

Paper: E-IIIT6
Algebraic Topology

Homotopy Theory : Fundamental Groups. Fundamental groups of Circle,Sphere and some surfaces. Geometrical construction of group structure on circle (in fact on any conic), Separation Theorem in the plane, Classification of surfaces. Simplical complex, Homology, Cohomology .

References :

1. Satya Deo ,Algebraic Topology-A Primer , Hindustan Book Agency
2. James r. Munkres, topology ,PHI
3. Anant R. Shastri, Basic Algebraic Topology, CRC Press Book.

Paper: E-IIIT7
Integral Equation and Integral Transform

Integral equations: classifications, successive approximations, separable kernels, Fredholm alternative, Hilbert-Schemidt theory of symmetric kernels, Construction of Green's function, Convoluted Kernels, Abels equations and solutions.
Calculus of Variations, Euler-Lagrange's equations, Geodesics, Minimum surface of revolution, Isoperimetric problems, Brachistochrone problem.

Integral transforms: Laplace and Fourier transforms, Applications to boundary Value Problems, Mellin & Hanckels transform, Inversion formulae, Bromwich Integral, Convolutions and applications, Distributions and their transforms. Applications to Wave, Heat and Laplace equations.

References:

1. M. Gelfand and S. V. Fomin. Calculus of Variations, Prentice Hall.
2. Linear Integral Equation: W.V. Lovitt (Dover).
3. Integral Equations, Porter and Stirling, Cambridge.
4. The Use of Integral Transform, I.n. Sneddon, Tata-McGrawHill, 1974
5. R. Churchil & J. Brown Fourier Series and Boundary Value Problems, McGraw-Hill, 1978
6. D. Powers, Boundary Value Problems Academic Press ,1979.

Paper: E-IIIT8
Differential Topology

We will study topological properties of smooth manifolds. Among topics that we will cover are:
Manifolds and Smooth Maps: Definitions, Derivatives and Tangents , The inverse function theorem, Immersions , Submersions , Transversality , Homotopy and Stability , Sard's theorem and Morse functions , Embedding Manifolos in Euclidean space.

Transversality and Intersection: Manifolds with boundary, one-manifolds and some consequences, Transversality, Intersection theory mod 2, Winding numbers and the Jordan-Brouwer separation theorem, The Borsuk-Ulam theorem.

Oriented Intersection Theory: Motivation, Orientation, Oriented intersection Number, Lefschetz fixed-point theory, vector fields and the Poincaré-Hopf theorem, the Hopf degree theorem , The Euler characteristic and Trjangulations.

References:

1. Topology from a differentiable viewpoint by Milnor.
2. Differential Topology by Guillemin-Pollack
3. Differential Topology by Hirsch
4. Topics in Differential Topology by Amiya Mukherjee

Paper: E-IIIT9
Theory of Approximation

Concept of Best Approximation in a Normed Linear Space,Existence of the Best Approximation, Uniqueness Problem, Convexity-Uniform Convexity, Strict Convexity and their relations. Continuity of the best Approximation Operator.
The Weierstrass Theorem, Bernstein Polynomials, Korovin Theorem, Algebraic and Trigonometric polynomials of the best Approximation. Lipschitz class, Modulus of Continuity, Integral Modulus of Continuity and their properties.

Bernstein's Inequality. Jacson's Theorems and their Converse Theorems.Approximation by means of Fourier Series.

Positive Linear Operators, Monotone Operators, Simultaneous Approximation, L^P - approximation. Approximation of analytic Functions.

References:

1. Mhaskar, H. M. and Pai, D. V., "Fundamentals of Approximation Theory", Narosa Publishing House.
2. Timan, A. F., "Theory of Approximation of Functions of a Real Variable", Dover Publication Inc.
3. E. W. Cheney, E. W., "Introduction to Approximation Theory", AMS Chelsea Publishing Co.
4. Lorentz, G. G., "Bernstein Polynomials", Chelsea Publishing Co.
5. Natanson, I. P., "Constructive Function Theory Volume-I", Fredrick Ungar Publishing Co.

Paper: E-IIIT10
p-adic Analysis

I. Congruences and modular equations

II. The p-adic norm and the p-adic numbers

III. Some elementary p-adic analysis

IV. The topology of Qp

V. p-adic algebraic number theory

<u>References:</u>

1. G. Bachman, Introduction to p-adic numbers and valuation theory, Academic Press (1964).
2. J. W. S. Cassels, Local fields, Cambridge University Press (1986).
3. F. Q. Gouvˆea, p-adic Numbers: An Introduction, 2nd edition, Springer-Verlag (1997).
4. S. Katok, p-adic analysis compared with real, American Mathematical Society (2007).
5. N. Koblitz, p-adic numbers, p-adic analysis and zeta functions, second edition, Springer-Verlag (1984).
6. S. Lang, Algebra, revised third edition, Springer-Verlag (2002).
7. K. Mahler, Introduction to p-adic numbers and their functions, second edition, Cambridge University Press (1981).
8. AM. Robert, A course in p-adic analysis, Springer-Verlag, 2000.

<u>Detailed Syllabus of PG Semester-IV</u>

Paper: E-IVT1
Signed Measure and Product Measure

Signed measure space. Decomposition of signed measures. Integration on a signed measure space.
Absolute continuity of a signed measure relative to a positive measure.

Radon- Nikodyn Theorem.

Product measure: Fubini's Theorem and Tonelli's Theorem.

<u>References:</u>

1. J. Yeh, Lectures on Real Analysis, World Scientific.
2. H. L. Royden, Real analysis, Macmillan Publishing Co., Inc. 4th Edition, 1993.
3. S. K. Berberian. Measure and integration. Chelsea Publishing Company, NY, 1965.
4. G. de Barra, Measure Theory and integration, Wiley Eastern Ltd. 1981.
5. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd. New Delhi.
6. Richard L. Wheeden and Antoni Zygmund, Measure and Integral: An Introduction to Real Analysis, Marcel Dekker Inc. 1977.
7. P.R. Halmos, Measure Theory. Van Nostrand. Princeton 1950.
8. K. R. Parthasarathy , Introduction to Probability and Measure, Macmillan Co. India Ltd.Delhi - 1977.
9. R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966.
10. Serge Lang. Aanlysis I & II, Addison-Wesley Publishing Co. Inc. 1967.
11. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi 1997

Paper: E-IVT2
Graph Theory

Preliminaries: Graphs,isomorphism, subgraphs, matrix representations, degree, operations on graphs, degree sequences .

Connected graphs and shortest paths: Walks, trails, paths, connected graphs, distance, cut-vertices, cut-edges, blocks, connectivity, weighted graphs, shortest path algorithms.

Trees: Characterizations, number of trees, minimum spanning trees .

Special classes of graphs: Bipartite graphs, line graphs, chordal graphs.

Eulerian graphs: Characterization, Fleury's algorithm, chinese-postman-problem.

Hamilton graphs: Necessary conditions and sufficient conditions.

Independent sets, coverings, matchings: Basic equations, matchings in bipartite graphs, perfect matchings, greedy and approximation algorithms .

Vertex colorings: Chromatic number and cliques, greedy coloring algorithm, coloring of chordal graphs,Brook's theorem.

Edge colorings: Gupta-Vizing theorem, Class-1 graphs and class-2 graphs, equitable edge-coloring .

Planar graphs: Basic concepts,Eulers formula, polyhedrons and planar graphs, charactrizations, planarity testing, 5-color-theorem .

Directed graphs: Out-degree, in-degree, connectivity, orientation,Eulerian directed graphs, Hamilton directed graphs, tournaments.

References:

1. J. P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw Hill Book Co. 1997
2. S. Witala, Discrete Mathematics - A Unified Approach, McGraw Hill Book Co.
3. C. L. Liu, Elements of Discrete Mathematics, McGraw Hill Book Co.
4. N. Deo. Graph Theory with Applications to Engineering and Computer Sciences, Prentice Hall of India.
5. D.B.West: Introduction to Graph Theory,Prentice-Hall of India/Pearson, 2009 (latest impression)
6. J.A.Bondy and U.S.R.Murty: Graph Theory and Applications (Freely downloadable from Bondy's website; Google-Bondy) .

Paper: E-IVT3
Topological Algebra

Topological rings and vector spaces: Definitions, examples, neighbourhood of zero, subring, ideals, quotients and projective limits of rings and vector spaces, completion of topological rings and vector spaces, Baire spaces, Summability, continuity of inversion, locally bounded vector spaces and rings, locally Retrobounded Division rings, Norms and Absolute values, Finite dimensional vector spaces, topological division rings, real valuations and valuation rings, discrete valuations, extensions of real valuations.

Reference:

1. Dikran Dikranjan, Introduction to topological groups.
2. Seth Warner, Topological Rings, Elsevier Science Publishers.
3. Ivan F. Wilde , Topological Vector Spaces (Lecture notes).

Paper: E-IVT4
General Theory of Integration

Tagged Gauge Partitions. Definitions, Cousins Theorem, Right-left Procedure, Straddle Lemma, Application in continuity, Intrinsic Power.

Henstock–Kurzweil Integral. Definition and basic properties. Fundamental Theorem, Saks-Henstock Lemma, Inclusion of the Lebesgue integral. SqueezTheorem, Vitali- Covering Theorem, Differentiation Theorem, Characterization Theorem.

Reference:

1. A Modern Theory of Integration, R. G. Bartle, AMS
2. Theories of Integration, Douglas S. Kurtz & Charles W. Swartz, World Scientific.
3. Lanzhou Lectures on Henstock Integration, Lee Peng Yee, World Scintific.
4. The Riemann, Lebesgue and General Riemann Integrals, A.G. Das, Narosa.
5. The general Theory of integration, R. Henstock, Clarendon Press.

Paper: E-IVT5
Modules and Number Theory

Module theory – Modules, sub modules, quotient modules; homomorphism and isomorphism theorems, Commutativity of Diagrams, Exact Sequences Four Lemma, Direct Sum, free modules, cyclic modules, simple modules. Fundamental Structure Theorem for finitely generated modules over a PID and its applications to finitely generated abelian groups.
Algebraic Number Theory - Algebraic number fields and the ring of integers, Embedding of a field in the field of Complex Numbers. Trace and norm of an element in a field, units and primes, factorization, quadratic and cyclotomic fields.

Characters of finite abelian groups, The Character Group, orthogonality relations for characters, Dirichlet characters.

Chebyshev's functions, Discussion of the Prime Number Theorem and its equivalent forms, Shapiro's Tauberian Theorem and its applications.

References:

1. *David S. Dummit and Richard M. Foote, Abstract Algebra (3e), John Wiley and Sons.*
2. *T. S. Blyth, Module Theory: An Approach to Linear Algebra, Oxford University Press, 1977.*
3. M. Atiyah, , I.G. MacDonald, *Introduction to Commutative Algebra*, Addison-Wesley, 1969.
4. *T.M. Apostol, Introduction to Analytic number theory, UTM, Springer, (1976).*
5. *Richard A. Mollin, Algebraic Number Theory, Chapman & Hall/CRC.*
6. *J. W. S Cassel, A. Frolich, Algebraic number theory, Cambridge.*
7. *M Ram Murty, Problems in analytic number theory, springer.*
8. *M Ram Murty and Jody Esmonde, Problems in algebraic number theory, springer.*

Paper: E-IVT6
Advanced Complex Analysis

Harmonic Function: Definition, Relation between Harmonic function and an analytic function, Examples, Harmonic Conjugate of a Harmonic function, Poisson's integral formula, Mean value property, The maximum and minimum principles, Dirichlet's problem for a disc and uniqueness of its solution, Characterization of harmonic function by mean value property.
Infinite Product: Definition, Necessary condition for convergence, General principle of convergence, Weierstrass inequality, Convergence of Infinite Product in terms of Corresponding series, Comparison test for Convergence, Absolute Convergence, Uniform convergence.

Integral Function: Factorization of Integral function, Weierstrass's Primary factor, Weierstrass factorization theorem, functions of finite order, Examples, The function n(r), Exponent of Convergence of Zeros, Canonical products, Hadamard's factorization theorem, Genus, Laguerre's Theorem, a-points of Integral function, Borel's Theorem, Picard's Theorem.

The Elementary Theory of Meromorphic Function: Poisson-Jensen formula, Characteristic function, First Fundamental Theorem, Cartan's Identity, Order of Growth, Comparative growth of $T(r)$ and $logM(r)$.

References:

1. H.A Priestly, Introduction to Complex Analysis, Clarendon Press Oxford, 1990.
2. J.B. Conway, Functions of one Complex variable, Springer-Verlag, International student-edition, Narosa Pub. House, 1980.
3. Liang-Shin Hahn and Bernard Epstein, Classical Complex Analysis, Jones and
4. Bartlett Pub. International London, 1996.
5. L.V. Ahlfors, Complex Analysis, McGraw.
6. S. Lang, Complex Analysis, Addison Wesley, 1970.
7. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
8. Mark J. Ablowitz and AS. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, South Asian edn. 1998.
9. E. Hille, Analytic Function Theory (2 Vols), Gonn and Co., 1959.
10. W.H.J. Fuchs, Topics in the Theory of Functions of one complex variable, D. Van Nostrand Co., 1967.
11. C. Caratheodory, Theory of Functions (2 vols), Chelsea Publishing Company,
12. 1964.

13. M. Heins, Complex Function Theory, Academic Press, 1968.
14. Walter Rudin, Real and Complex Analysis, McGraw- Hill Book Co., 1966.
15. S. Saks and A Zygmund, Analytic Functions, Monographie Matematyczne, 1952.
16. E.C. Titchmarsh, The Theory of Functions, Oxford University Press, London.
17. W.A Veech, A Second course in Complex Analysis, WA. Benjamin, 1967.
18. S. Ponnusamy, Foundations of Complex Analysis,Narosa Publication House, 1997 **.**

Paper: E-IVT7
Discrete Mathematics

Number Theory and Cryptography: Divisibility and Modular Arithmetic, Integer, Representations and Algorithms, Primes and Greatest**,** Common Divisors, Solving Congruences, Applications of Congruences, Cryptography.
Counting Techniques: The Basics of Counting, The Pigeonhole Principle Permutations and Combinations, Binomial Coefficients and Identities, Generalized Permutations and Combinations, Applications of Recurrence Relations, Solving Linear Recurrence Relations, Recurrence Relations, Generating**,** Functions, Principle of Inclusion–Exclusion, Applications of Inclusion–Exclusion. Modeling with recurrence relations with examples of Fibonacci numbers and the tower of Hanoi problem, Solving recurrence relations. Divide-and-Conquer relations with examples (no theorems).Generating functions, definition with examples, solving recurrence relations using generating functions, exponential generating functions. Difference equations.

Order Relations and Structures: Product Sets and Partitions, Relations, Properties of Relations, Equivalence Relations, Partially Ordered Sets, Extremal Elements of Partially Ordered Sets, Lattices, Finite Boolean Algebras, Functions on Boolean Algebras, Boolean Functions as Boolean Polynomials. Definition and types of relations. Representing relations using matrices and digraphs, Closures of relations, Paths in digraphs, Transitive closures, Warshall's Algorithm.

Groups and Coding Theory: Binary Operations Revisited, Semigroups, Products and Quotients of Semigroups, Groups, Products and Quotients of Groups, Coding of Binary Information and Error Detection, Decoding and Error Correction.

References:

1. Kenneth H. Rosen - Discrete Mathematics and Its Applications, Tata Mc-Graw-Hill, 7th Edition, 2012.
2. Bernard Kolman, Robert C. Busby, Sharon Cutler Ross-Discrete Mathematical Structures-Prentice Hall, 3rd Edition, 1996.

3. Grimaldi R-Discrete and Combinatorial Mathematics. 1-Pearson, Addison Wesley, 5th Edition, 2004.
4. C. L. Liu – Elements of Discrete Mathematics, McGraw-Hill, 1986.
5. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.
6. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
7. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi, 1994.
8. G. Chartand and L. Lesniak – Graphs and Diagraphs, wadsworth and Brooks, 2nd Ed.,
9. Clark and D. A. Holton – A First Look at Graph Theory, Allied publishers.
10. D. B. West – Introduction to Graph Theory, Pearson Education Inc.,2001, 2nd Ed.,
11. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier, 1976.

Paper: E-IVT8
Algebraic Geometry

Part I. Preliminaries: Some category theory, Motivation, Categories and functors, Universal properties determine an object up to unique isomorphism, Limits and colimits, Adjoints, An introduction to abelian categories, Spectral sequences, Sheaves. Motivating example: The sheaf of differentiable functions. Definition of sheaf and presheaf, Morphisms of presheaves and sheaves, Properties determined at the level of stalks, and sheafification, Sheaves of abelian groups, and OX-modules, form abelian categories, The inverse image sheaf, Recovering sheaves from a “sheaf on a base”.

Part II. Schemes: Toward affine schemes: the underlying set, and topological space, Toward schemes, The underlying set of affine schemes, Visualizing schemes I: generic points, The underlying topological space of an affine scheme, A base of the Zariski topology on Spec A: Distinguished open sets, Topological (and Noetherian) properties, The function I(·), taking subsets of Spec A to ideals of A, The structure sheaf, and the definition of schemes in general, The structure sheaf of an affine scheme, Visualizing schemes II: nilpotents, Definition of schemes , Three examples, Projective schemes, and the Proj construction, Some properties of schemes, Topological properties, Reducedness and integrality, Properties of schemes that can be checked “affine-locally”, Normality and factoriality, Where functions are supported: Associated points of schemes.

Part III. Morphisms: Morphisms of schemes, Introduction, Morphisms of ringed spaces, From locally ringed spaces to morphisms of schemes, Maps of graded rings and maps of projective schemes, Rational maps from reduced schemes, Representable functors and group schemes, The Grassmannian (initial construction), Useful classes of morphisms of schemes, An example of a reasonable class of morphisms: Open embeddings, Algebraic interlude: Lying Over and Nakayama, A gazillion finiteness conditions on morphisms, Images of morphisms: Chevalley's theorem and elimination theory, Closed embeddings and related notions, Closed embeddings and closed subschemes, More projective geometry, Smallest closed subschemes such that ... , Effective Cartier divisors, regular sequences and regular embeddings, Fibered products of schemes, and base change, They exist, Computing fibered products in practice, Interpretations: Pulling back families, and fibers of morphisms, Properties preserved by base change, Properties not preserved by base change, and how to fix them, Products of projective schemes: The Segre embedding, Normalization, Separated and proper morphisms, and (finally!) varieties, Separated morphisms (and quasiseparatedness done properly), Rational maps to separated schemes, Proper morphisms.

Part IV. "Geometric" properties: Dimension and smoothness: Dimension, Dimension and codimension , Dimension, transcendence degree, and Noether normalization, Codimension one miracles: Krull's and Hartogs's Theorems, Dimensions of fibers of morphisms of varieties, Proof of Krull's Principal Ideal and Height Theorems, Regularity and smoothness, The Zariski tangent space, Regularity, and smoothness over a field, Examples, Bertini's Theorem, Discrete valuation rings: Dimension, Noetherian regular local rings, Smooth (and etale) morphisms (first definition), Valuative criteria for separatedness and properness, More sophisticated facts about regular, local rings, Filtered rings and modules, and the Artin-Rees Lemma.

References:

1. Atiyah and MacDonald 1969: Introduction to Commutative Algebra, Addison-Wesley.
2. Milne, J.S., Commutative Algebra, v4.02, 2017.
3. Milne, J.S., Fields and Galois Theory, v4.52, 2017.
4. Hartshorne 1977: Algebraic Geometry, Springer.
5. Shafarevich 1994: Basic Algebraic Geometry, Springer.

Paper: E-IVT9
Category Theory

Categories 1 Introduction, Functions of sets, Definition of a category, Examples of categories, Isomorphisms, Constructions on categories, Free categories, Foundations: large, small, and locally small

Abstract structures: Epis and monos, Initial and terminal objects, Generalized elements, Sections and retractions, Products, Examples of products, Categories with products, Hom-sets

Duality: The duality principle, Coproducts, Equalizers, Coequalizers

Groups and categories: Groups in a category, The category of groups, Groups as categories, Finitely presented categories

Limits and colimits:

Subobjects, Pullbacks, Properties of pullbacks, Limits, Preservation of limits, Colimits

Exponentials: Exponential in a category, Cartesian closed categories, Heyting algebras, Equational definition, λ-calculus, Exercises

Functors and naturality: Category of categories, Representable structure, Stone duality, Naturality, Examples of natural transformations, Exponentials of categories, Functor categories, Equivalence of categories, Examples of equivalence

Referencess:

1. Category Theory by Steve Awodey.
2. Categories for the working Mathematician by Saunders Mac Lane
3. An Introduction to Category Theory by Harold Simmons.
4. Basic Category Theory by Tom Leinster.

Paper:S-IV6P
Numerical problem solving by computer Programming (PRACTICAL)

Group A

C Programming

An overview of computer programming languages – modular programming and program development cycle. Character set, keywords and identifiers; Variables and Constants; Fundamental data types – int, short, long; float, double; char; type conversion and casting; Operators and Expressions – arithmetic operators, relational operators, logical operators, assignment operators, increment and decrement operators, bitwise manipulation operators, size of operator, conditional operator; operator precedence and associativity; void data type.

Conditional Branching – if, if-else, switch; Looping and nested looping – for, while, do-while; break and continue, goto; Infinite loops, Header file and include directive, macro substitution and conditional compilation, scanf, printf and various format specifiers, Standard C library functions.

Declaring, initializing and using arrays in programs; Arrays and memory; One dimensional and multidimensional arrays; Character arrays and strings. Pointer arithmetic; Accessing array elements through pointers; Arrays of pointers; Pointers to pointers; Sorting algorithms. Passing arguments to a function, declaring and calling a function; Pointers to functions; Passing arrays as function arguments; Recursion; main() function. Opening and closing a file; Reading from a file and writing to a file; Random access and error handling.

Group- B

Solving Numerical Problems using C – Programming

1. Interpolation: Newtown forward, Newtown backward, Stirling, Lagrange etc.
2. Differentiation: Using interpolated polynomials.
3. Integration: Trapizoidal Method, Simpson Method, Romberge Method, Gauss Quadrature Method.
4. Matrix inversion: Gauss Jordan method.
5. Largest Eigen value and corresponding eigen vector of a squre matrix: Power method.
6. System of Linear equation: Gauss Elimination method, Gauss Seidal method.
7. O.D.E. : Milnes method, Adams method.
8. P.D.E. : Prarabolic, Laplace, Hyperbolic.

- Note Book + Viva-voce.

Reference:

1. B. Gottfried: Programming with C , Tata McGraw-Hill Edition 2002.
2. E. Balagurusamy : Programming in ANSI C, Tata Mcgraw Hill - Edition 2002.
3. Brain W. Kernighan & Dennis M. Ritchie, The C Programme Language, 2nd Edition (ANSI features) , Prentice Hall 1989.
4. Let Us C- Y.P. Kanetkar, BPB Publication - 2002.
5. Analysis of Numerical Methods—Isacsons & Keller.
6. Numerical solutions of Ord. Diff. Equations—M K Jain
7. Numerical solutions of Partial Diff. Equations—G D Smith.
8. Programming with C, B. Gottfried, Tata-McGraw Hill
9. Programming with C, K. R. Venugopal and Sudeep R. Prasad, Tata-McGraw Hill